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Identical service and the odd or even transform of Laplace

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Abstract. The article is devoted to two joint work assignments with the professor A.D.Solovjev. In the first part we consider K nodes in series in conditions of identical service: the time of service of one customer on one device of nodes is the same for all different nodes (a.s.). In every node N devices are located (N more 0). Each node contains the infinite set of places for wait. The total number of customers (in real situation), the time of wait, the total time of work in nodes $2, ..., K$ remains limited for all moments of times in heavy traffic conditions on the first node, if the classical loading is equal to 1 or more 1 or not more 1. The fact takes place, if the time of service of one customer by one device of node is limited by the constant. In the second part of the article a new class of the odd or even transforms of Laplace is presented. The class leads to some unforeseeable consequences in the direction of the Fourier transforms. We have proved a regularity of the transforms of Laplace in $|z| < b > 0$ for a new class of functions. The regularity leads to some results in direction of a new inversion of Laplace transform only with help of positive values of the transform on real axis.

Keywords: Identical service for multi-channels nodes in series, heavy traffic, total time of work, waiting times, transform of Laplace, transform of Fourier, evenness of the transform of Laplace, new inverse of the transform of Laplace.

1. Introduction

The article is devoted to two joint work assignments with the professor A.D.Solovjev. A primary aim of the first part of work (the theorem 1) is a generalization of the [1–3] results on some general nets with the total loading $\rho > 0$ (on input nodes) in conditions of identical service, [4, 5]. The sharp reduction of time of expectation and total work on the inlying nodes was marked, [3–5] (for identical service). The leading role at such reduction is played by the identity of Legendre (the first the identity for the identical service appeared in works of author, [3–5]):

$$f(Y(t) - f(Y(t))) \equiv 0, t \in [0, 1],$$

for any continuous on $[0, 1]$ functions $Y(t), Y(0) = 0$, where the transformations of Legendre is (by definition)

$$f(Y(t)) = Y(t) - t - \inf_{u\in[0,t]} (Y(u) - u) = \sup_{0\leq u\leq t} (Y(t) - Y(u) - (t - u)).$$

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Some applications of the given results are resulted in the examples 1,2.

In the second part of the article we consider a new results of the author of given article devoted to the double transform of Laplace (lemma 1), [5, 7]. A regularity of such transform in the area of 0 results in many new theorems, related to the inverse problem of the cosine-sine transform of Fourier and Laplace (and for the new class of odd or even transform of Laplace in the lemma 2 in the article), [5, 7].

2. Identical service; the regularity of the transform of Laplace in the area of 0

Main part of the first part is expounded in the examples 1-2. The examples follow from the [3, 5] works of author.

We consider the K nodes, $K \geq 2$. The arrival process for the j-th node is equal to the output process on the $(j-1)$-th node, $j = 2, \ldots K$. In the first node the customers are served in order of arrival, and as soon as the service is finished the customer arrives on the next node. In all nodes the customers are served in order of arrival. If a customers arrives on a node in group (for non-ordinary process), the customers are disposed in the group in the random order - all results d't rely on order in the group. The j-th node consists of N service units (devices) with infinite set of waiting places, $j = 1, \ldots K$.

In our article for $j > 1$ we explore a net characteristics: W_j^t - the total time of service of all the customers being on j-th node at t moment, $j = 1, \ldots, K$ (the virtual time of wait for one channel); ν_j^t - the total number of the customers on j-th node at t moment, $j = 1, \ldots, K$; V_j^t - the full time of service on the j-th node of the customer, which arrives on the j-th node at t moment $j = 1, \ldots, K$ (the time of wait plus the "length of customer").

By definition, $A(t)$ is a number of final customer among all customers arriving on first node during $[0, t]$.

Let j be the number of the customer arriving on the first node j-th on the account. By definition, ξ_j^i is the time of service of the j-th customer on the i-th node by one device (unit of service); **all the different devices on all nodes are identical:**

$$\xi_j^1 = \xi_j^2 = \ldots = \xi_j^K,$$

for all $j = 1, 2, \ldots$, and $\{\xi_j^1, j = 1, 2, \ldots\}$ are the mutually independent random values with the distribution function $F(x) = \Pr(\xi_j^1 \leq x)$ for all T.

The process $A(t), 0 \leq t \leq 1$, and the sequence $\{\xi_j^1, j = 1, 2, \ldots\}$ are mutually independent for all $T \in [0, \infty)$.

Example 1. Let's consider K consistently located devices with **identical** service $(N = 1)$. If the time of service of one customer by one device more of m constant, and it is always limited by $2m$ constant:

$$0 < m < \xi_1^1 < 2m < \infty,$$

the number of customers on every devices with numbers $j > 1$ **will be or** 0 **or one or two** (independently from an intensity of arriving of customers on the first device), if $\rho < 1, \rho = 1, \rho > 1$, where ρ is the traditional loading on the first device: $\nu_j^t \leq 2$, for all $t \in [0, \infty), j = 2, \dots, K$, if $\nu_j^0 = 0, j = 1, \dots, K$. The situation (the theorem 1) is executed for all input streams of customers on first device ([5]).

For instance,the number of customers on every devices **with numbers** $j > 1$ **will be or** 0 **or one or two**, for all known input process and for all ρ, if

$$m < \xi_1^1 < 2m,$$

where ξ_1^1 is the length of service of one customer.

Example 2. We consider K consistently located devices with **identical** service ($N = 1$) (one device in one node). The number of customers **on every devices with numbers** $j > 1$ **will be or** 0 **or one (not more), if the time of service of every customer by one device is the** m **constant for all nodes** :

$$\nu_j^t \leq 1,$$

for all $t \in [0, \infty), j = 2, \dots, K$, if $\nu_j^0 = 0, j = 1, \dots, K$,

$$\xi_j^1 \equiv m = const < \infty, \quad j = 1, 2, \dots,$$

[5].

As in the first example the situation (the theorem 1) is executed for all input streams of customers on first device and for all $\lambda m < 1, \lambda m = 1, \lambda m > 1$, for instance $m = 0,1$, $\lambda = 0,0001$ or $\lambda = 10$ or, $\lambda = 1000$.

Theorem 1. [3–5]

1.

$$\max_{i=2,\dots,K} \sup_{0 \leq t \leq T} \max(W_i^t, V_i^t) \leq C(N)\Delta,$$

$\Delta(T) = \max_{j \leq A(T)} \xi_j^1, C(N) = 2N - 1$; and $C(N) = 1$,if $N = 1$, $C(N) = 2N - 1$, if $N > 1$. (For all the input processes $A(t)$).

By definition,

$$L_{\pm}Z(t)(\cdot)(x)=\int_0^\infty e^{\pm xt}Z(t)dt,\quad x\in[0,\infty),\quad L_+=L;$$

$$CoS(t)(\cdot)(x)=\int_0^\infty \cos xtS(t)dt,\quad SiS(v)(\cdot)(x)=\int_0^\infty \sin xtS(t)dt,$$

$$x\in(-\infty,\infty);$$

$$F_{\pm}S(t)(\cdot)(p)=\int_{-\infty}^\infty e^{\pm pit}S(t)dt,\quad F_{\pm}^0S(t)(\cdot)(p)=\int_0^\infty e^{\pm pit}S(t)dt,$$

$$p\in(-\infty,\infty).$$

Lemma 1([5]).
The

$$R_+(z)=\int_0^\infty e^{zt}dt\int_0^\infty e^{itx}S_0(x-a)dx,\ LLS_0(x-a)(\cdot)(z),$$

functions are regular in the area $z:|z|<2a>0$, if

1. the function $S_0(-z)$ is regular in the area $G(S)=\{z:|Im\,z|<2a\}$;
2. $|S_0(z)|<c_0/|z|^{2+\delta},|z|\to\infty,z\in G(S),\delta>0,\ c_0,\delta=const.$ (We do not use $S_0(0)=0$).

Proof.
In the integral

$$\int_0^\infty e^{pt}dt\int_{-\infty}^{-a} e^{txi}S_0(x)dx=\int_0^\infty e^{pt}dt\int_{-\infty}^\infty e^{txi}S_0(x)dx-$$
$$-\int_0^\infty e^{pt}dt\int_{-a}^\infty e^{txi}S_0(x)dx=L_+F_+S_0(x)(\cdot)(p)+$$
$$+\left[-\int_0^\infty e^{(p-ai)t}dt\int_0^\infty e^{txi}S_0(x-a)dx\right],$$

the first part of the sum is regular in the $|Re\,p|<2a$ area, if the $S_0(-z)$ function is regular in $|Im\,p|<2a$ (the fact is well-known, see [5, 7] or many works of Privalov).

We obtain, that the first part of the sum is regular in $\{|Re\,p|<2a\}=G_1(s)$, where $\{z:|z|<2a>0\}\in G_1(s)$. The second part of the sum together with the integral $J(p) = \int\limits_0^{\infty} e^{pt} dt \int\limits_{-\infty}^{-a} e^{txi} S_0(x) dx = \int\limits_{-\infty}^{-a} [-S_0(x)/(p+ix)] dx$ is regular for all $p : Im\, p \in (-a, +\infty), a > 0$ (the integral obviously has $dJ(p)/dp$ in the area ([6, 7]) .

We obtain, that the second part of the sum $[-\int\limits_0^{\infty} e^{(p-ai)t} dt \int\limits_0^{\infty} e^{txi} S_0(x-a) dx] = J_0(p)$, is regular for all $z = p - ai : Im\, p \in (-a, +\infty) \bigcap \{|Re\, p| < 2a\}$, or for the all $G_0 = \{z : Im\, z \in (-2a, +\infty) \bigcap \{|Re\, z| < 2a\}\}$, where $|z| < 2a \in G_0$. The lemma 1 we obtain now from $LLS_0(x-a)(\cdot)(iu) = (-i)R_+(-u), u \in (0, +\infty)$, [5].

From the lemma 1 we get the obvious **Lemma 2**:

In the conditions of the lemma 1 $R_1(-p) = -R_1(p), p \in C$, where $R_1(p) = LCo(S(x))(\cdot)(p), S(-x) = S(x)$, $x \in (-\infty, \infty)$; $R_2(-p) = R_2(p)$, $p \in C$, where $R_2(p) = LSi(S(x))(\cdot)(p), S(-x) = -S(x)$, $x \in (-\infty, \infty)$, [5], (the values on the complex axis do not have the real part).

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