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Industrial revolution and reform of mathematics

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Abstract. The industrial revolution led to changes in the structure of the community of mathematicians. These changes resulted in significant changes in the structure of mathematics.

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Industrial Revolution. The Industrial Revolution was the transition to new manufacturing processes in the period from about 1760 to sometime between 1820 and 1840. This transition included going from manual production methods to machines, new chemical manufacturing and iron production processes, improved efficiency of water power, the increasing use of steam power, the development of machine tools and the rise of the factory system. The infrastructure (transport, banking, interregional and international commerce) also developed rapidly.

New plants and factories, new machines and energy sources needed qualified workers, engineers and technicians who could service complex mechanisms. Naturally, educational institutions for the training of qualified workers, technicians and engineers were necessary.

Education and Industrial Revolution. Before the Industrial Revolution, good education was available to a very small number of people. Systematic education was, generally, theological. The students of the Universities studied seven "liberal arts" including arithmetic and geometry. High-quality education was actually individual. The majority of the population was illiterate.

But the development of industry and transport required educated people. Qualified workers needed to know mathematics, they had to be able to read the drawings and serve complicated machinery; their technical education was based on mathematics.

And at the end of the 18th century and the beginning of the 19th century, we see the emergence of a large number of primary schools in Europe and in America, and the development of middle schools is also beginning. In the 19th century, many new higher technical schools were set up. A graduate of a secondary school could be an elementary school teacher. A graduate of a University could be a secondary school teacher, and, after additional training, he could be professor of higher technical school. So, there was need for mathematics teachers, and theirs education. Thus, the quantity of the professors of mathematics increased.

The structure of the community of mathematicians began to change.

Structure of community of mathematicians. From ancient times two intersecting groups of mathematicians existed: teachers and practicians. We see the traces of their activities in all ancient civilizations. For a long time, the mathematicians solved practical problems. All mathematical knowledge was associated with the surrounding environment. The ancient mathematics operated with concepts having origins in everyday practice. The first abstract mathematical problems arise in the Arab medieval mathematics. Basically, it was algebraic equations of a high degree and 5th postulate study. So, in the Arab East a new small group of abstract mathematicians (*"pure theorists"*) appeared. However, these *theorists* also solved the practical problems of mathematics.

Renaissance brought Greek knowledge back to Europe, but with the help of Arab science. Therefore, in Europe the interest to the abstract problems (generally, in algebra and number theory) was formed. The time of the chivalry ended, and now the gentlemen sought to attract the attention of ladies through the intellect (and, often, through the costume or the wealth). Interesting mathematical problems became very popular among the nobility. A new group of mathematicians was formed: *the amateurs of mathematics*. They invented and solved new problems in algebra and number theory. In addition, some teachers of mathematics invented new algebraic and geometric problems for training schoolchildren; it is possible to recognize part of the teachers as "*the amateurs of mathematics*".

So, shortly before the Industrial Revolution, the community of mathematicians consisted of teachers, practitioners, and amateurs. Most of the amateurs were "theoreticians". However, some teachers and practitioners were also interested in "theoretical" problems, i.e. problems that did not have direct applications in human activity.

Many amateurs of mathematics have made a significant contribution to the development of this science. Perhaps because they were not limited in their thinking by some standards.

Note that at this time the University professors were the teachers and the practicians: European Universities and Academies fulfilled state requirements on mathematical support of important areas of state activity (navigation, design and creation of new transport routes, improvement of weapons, calculation of financial flows, etc.). Also, they researched some of the popular abstract problems, for example, in the theory of numbers and algebra.

Let us recall that each professor had to do research and publish the results of his studies: this was a reason for the position of the professor.

Until the end of the 18th century, University professors had a sufficient number of practical problems for research.

Thus, by the beginning of the 19th century, the mathematical community consisted of intersecting groups: teachers, practicians, theorists, amateurs.

Most theorists were amateurs. Basically, they invented and solved problems in algebra and number theory.

Most University professors solved practical problems: they created a mathematical model of the real situation and studied its behavior. Many practical problems were connected with the definition of the characteristics of technical, physical and other systems, as well as the optimization. Accordingly, the problems of differential equations, calculus of variations, statistics, and computational methods were very popular.

19th century: professoriate. So, at the beginning of the 19th century the number of schools for children increased, the number of teachers increased correspondingly, and many new higher educational institutions appeared for the training of secondary school teachers, the engineers and the technicians. Naturally, the number of professors of mathematics increased.

But there were not enough practical problems to ensure the scientific work of all professors-mathematicians. Many of them began to study the so-called theoretical questions in mathematics.

It was such topics: the ordering and structuring of mathematical knowledge; the generalization of some mathematical problems and notions; arbitrary change of parameters of previously studied problems; etc.

Ordering and structuring of mathematical knowledge. The rapid development of the system of technical education required the solution of the methodological problems of mathematics, because:

1. Successfully formulated concepts and convenient notation simplified further research;

2. The unification and the adaptation of mathematical concepts and results for higher school students (mainly technical schools) were required for the successful teaching of mathematics.

Many mathematical notions did not have strict definitions. The mathematicians used some intuitive concepts for the objects such as function, limit, calculus of infinitesimal and others. Many proofs and logical reasoning were not strict. However, in the solution of applied problems arising from practical activities such non-stringent constructions did not lead to significant errors.

But, for the teaching a large number of students, the teachers needed some structure that would united the widely developed body of mathematics. The mathematicians of the 19th century began to create a structure of new mathematics on the model of the Euclid's "Elements".

Cauchy introduced some criteria of rigour for the proof in the mathematical analysis; this work was continued by the great educator Weierstrass. We know that Weierstrass, Cauchy, Heine and Bolzano formalized the notion of limit and infinitesimal, continuity, etc. Then Weierstrass, Dedekind and Cantor offered answers to the question: "What is the number?".

Attempts to create a universal logical mathematical language and notions for the inference of all possible mathematical facts were undertaken. This led to the creation and development of mathematical logic and set theory. The foundations of the modern ideology of teaching higher mathematics were created at this time.

Almost all the mathematics reformers were University professors; their desire to construct a rigorous logical structure of mathematics would replace the intuitive ideas of mathematical problems solutions by strong logical structures. The theorizing professors prepared the students-theorists; the statement "*mathematics=logic*" become a symbol of the University mathematics of the second half of the XIX century.

Generalizations. Here are some reasons for the appearance of the generalizations.

1. New objects of the study in applied problems appeared. For example, at first the functions were only polynomials; then the polynomials and trigonometrical functions, then they added a logarithm and an exponent, and later the notion of the function was generalized – a first by L. Euler, then by B. Bolzano, L. Dirichlet and N. Lobachevsky. But the generalized definition of the function generated many unaccustomed functions, such as the Dirichlet function and the Riemann function. The ability to define a function as the limit of a sequence of functions led to physically impossible ones, for example the Cantor function or the Devil's staircase.

New and unaccustomed mathematical objects did not possess the properties of the usual mathematical concepts associated with practical applications. Many intuitive assumptions about their properties were false. Therefore, many *counterexamples* appeared as an important part of the mathematical verification of new hypotheses.

2. A generalization allows everyone to create a universal proof for many theorems or a universal method for solving many problems. For example, some 1D, 2D and 3D problems have similar solutions; thus, it can propose a concept of multidimensional (Euclidean) space, and formulate a generalized n-dimensional problem with a generalized solution. So, the generalization can give the useful method for proof, but arbitrary generalization can lead to paradox or to the problem without solution (as Fermat's Last Theorem).

An important step in the development of abstract mathematical knowledge is the generalization of algebraic concepts and their application in various areas of analysis, geometry and differential equations theory.

Changing conditions of mathematical problems. In the 19th century, many mathematicians did not participate in the experimental and technical studies. They were given the mathematical problems formulated by other researchers. Not knowing the reasons for the emergence of a particular problem, the mathematician would generalize this problem, introducing additional conditions and extensions of used concepts. There was a view that all mathematical problems are the product of human thought, and not the product of human practical activity.

Generally speaking, if the mathematical problem is correctly formulated, and has the origin in the real processes of the nature, then this problem always has a unique solution, which describes the original process in the nature.

But when mathematicians tried to solve the generalized problem with arbitrarily changed parameters, they found that this generalized problem did not always have the solution. So, in the 19th century a new topic appeared in mathematical research: *existence and uniqueness theorem.*

Discussion of status and future of mathematics. Increased mathematical knowledge needed to organize and identify the outlook for its development.

In 1872 Felix Klein proposed a method of classifying geometries by their underlying symmetry groups (*Erlangen program*). Naturally, algebraization of geometry facilitated the solution of many problems, but such methods translated geometric facts into other algebraic language, and the essence of geometry began to feel a kind of abstraction.

Later, the outstanding theorists F. Klein and G. Cantor initiated the international congresses of mathematicians. In 1897, at the Congress in Zurich, the question of the justification of mathematics based on set theory and problems of mathematical education were discussed. However, set theory was fraught with many paradoxes.

Questions about the justification of mathematics caused a lot of controversy. Many mathematicians expressed their own views on principles for the development of mathematics.

One of those ways was the formalization of mathematical knowledge and proofs of theorems based on logical structures.

Another variant of development of mathematics is a constructive mathematics, i.e. the mathematical proof of existence of the solution is an algorithm of constructing this solution, without the law of the excluded middle.

The third proposed way of development of mathematics is intuitionism, where mathematics is considered to be purely the result of the constructive mental activity of humans rather than the discovery of fundamental principles claimed to exist in an objective reality.

Before the Industrial Revolution, almost all research in mathematics was constructive. Mathematicians solved some of the problems and found certain solutions. In some cases of similar problems they have developed a general method to obtain the solution.

New abstract mathematics proved the existence of a solution, but not always gave algorithm to obtain it.

The completion of the first step in the reform of mathematics was the work of D. Hilbert.

Then there was A. Whitehead, B. Russell, N. Bourbaki, etc. Without reform of mathematics occurred due to the Industrial Revolution, modern mathematics would have been different than the one we have now.

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