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On Reliability Function of a Parallel System with Three Renewable Components

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Abstract. Considered system consists of three renewable components that are connected in parallel. The components are described by continuous time independent alternating processes. The sojourn times in the operative state for all components have exponential distributions. The sojourn times in the failed state have arbitrary absolute continuous distributions. All sojourn times are independent. The system is worked at time t if at least one component is worked. It is necessary to calculate system reliability on given time interval for the known initial states of the components. Non-stationary and stationary regimes are considered.

Keywords: alternating processes, recurrent event, renewal equation, system reliability.

1. Introduction

Consideration of a system reliability function is one of main problems in reliability theory. Simple redundant renewable systems were firstly objects of investigations. Homogeneous cold standby system has been considered in the book of Gnedenko and all [1]. The case of hot standby and two components discussed in the paper [2]. The similar problems have been studied also by Rykov et al. [3 - 5] with the help of Markovization method and Laplace transformation.

In this paper the three-component system with a hot standby is considered. The components are described by continuous time alternating processes $X_1(t), X_2(t)$ and $X_3(t)$. These processes are independent. The sojourn times in the state 0 (up state) of all processes have exponential distributions with parameters λ_1, λ_2 and λ_3. The sojourn times in the state 1 (down state) have nonnegative distributions with probability density functions (p.d.f.) $\alpha_1(t), \alpha_2(t)$ and $\alpha_3(t)$. All sojourn times are independent. The system is working at time t if at least one of components is working. Thus the integrated state of the system $Z(t) \in \{0, 1\}$ can be represented as

$Z(t) = X_1(t) \wedge X_2(t) \wedge X_3(t)$. It is necessary to calculate system reliability on interval $(0, t)$:

$$R(t) = \mathbf{P}\{Z(\tau) = 0 : \tau \in (0, t) \,|\, X_1(0) = X_2(0) = X_3(0) = 0\}. \quad (1)$$

Peculiarity of this paper consists in using the renewal theory for considered problem solution. It allows to receive an explicit expression for the reliability function of a three component system with a hot standby.

2. Reliability function

Let us consider the first time t, when the system comes to the state $(X_1(t) = 0, X_2(t) = 0, X_3(t) = 0)$ from any other state at that $Z(\tau) = 0 :$ $\tau \in (0, t)$. We say that *a recurrent event* occurs at this time.

The three dimensional process $X(t) = (X_1(t), X_2(t), X_3(t))$ has 8 states that can be numbered as

$$0 = (000), 1 = (001), etc., 6 = (110), 7 = (111).$$

State 7 corresponds to the failure; state 0 corresponds to the system state, when all components are working. For the process investigation we will use the additional variables method, namely for any component of the process being in the down state, $X_i(t) = 1$, an additional variable x will be used, which means the elapsed time in this state for this component.

Let for indices $i, j \in \{1, 2, 3\}$, $i \neq j$, the function $\varphi(i, j) \in \{1, 2, 3\}$ be such that $\{1, 2, 3\} = \{i, j, \varphi(i, j)\}$.

Further we denote $\lambda_{\sum} = \lambda_1 + \lambda_2 + \lambda_3$, $\lambda_{(j)} = \lambda_{\sum} - \lambda_j$.

Now we propose equations for the p.d.f. $f(t)$ of the first recurrence event time occurrence for the initial state with all working components. A notation $f_i(t, x)$ instead of $f(t)$ is used, if at initial time $t_0 = 0$ the i-th component has been in failed state during time $x > 0$. We will use the notation $f_{i,j}(t,\ x)$ for this density if at initial time $t_0 = 0$ the i-th component has been failed during time x and the j-th component fails just now. Further let $A_i(t)$ and $\bar{A}_i(t)$ be the cumulative distribution function (c.d.f.) of the repair time and its supplement:

$$A_i(t) = \int\limits_0^t \alpha_i(\tau) d\tau, \quad \bar{A}_i(t) = 1 - A_i(t), \quad t \geq 0, \quad i = 1, 2, 3.$$

Then the following expressions for the introduced functions can be obtained with help of complete probability formula:

$$f_{i,j}(t,x) = \frac{1}{\bar{A}_i(x)} \int\limits_0^t \exp(-u\lambda_{\varphi(i,j)}) \left[\bar{A}_j(u)\alpha_i(x+u) f_j(t-u,u) + \right.$$

$$\left. + \bar{A}_i(x+u)\alpha_j(u) f_i(t-u,x+u)\right] du,$$

$$f_i(t,x) = \exp(-t\lambda_{(i)}) \frac{1}{\bar{A}_i(x)} \alpha_i(t+x) +$$

$$+ \frac{1}{\bar{A}_i(x)} \int\limits_0^t \exp(-u\lambda_{(i)}) \bar{A}_i(x+u) \sum_{j \neq i} \lambda_j f_{i,j}(t-u,x+u) du.$$

Finally:

$$f(t) = \int\limits_0^t \exp(-u\lambda_{\sum}) \sum_{j=1}^{3} \lambda_j f_j(t-u,0) du.$$

Let $G_i(t,x)$ be the conditional probability that during the interval $(0,t)$ there was neither system failure, nor recurrent event, given at the time $t_0 = 0$ the i-th component has been failed during time $x > 0$. Let $G_{i,j}(t,x)$ be the analogous probability, under condition that in the initial time moment 0 the i-th component is failed during times x and the j-th component fails just now. Then

$$G_{i,j}(t,x) = \exp(-t\lambda_{\varphi(i,j)}) \frac{\bar{A}_i(x+t)}{\bar{A}_i(x)} \bar{A}_j(t) +$$

$$+ \frac{1}{\bar{A}_i(x)} \int\limits_0^t \exp(-u\lambda_{\varphi(i,j)}) (\bar{A}_j(u)\alpha_i(x+u) G_j(t-u,u) +$$

$$+ \bar{A}_i(x+u)\alpha_j(u) G_i(t-u,x+u)) du.$$

$$G_i(t,x) = \frac{\bar{A}_i(x+t)}{\bar{A}_i(x)} \exp(-\lambda_{(i)} u) + \frac{1}{\bar{A}_i(x)} \int\limits_0^t \exp(-\lambda_{(i)} u) \times$$

$$\times \bar{A}_i(x+u) \Big(\sum_{j \neq 1} \lambda_i G_{i,j}(t-u,x+u)\Big) du.$$

Probability that during the interval $(0, t)$ there was neither system failure, nor recurrent event, under condition that at time $t_0 = 0$ all components have been working, has the form:

$$H(t) = \exp(-\lambda_{\sum} t) + \int_0^t \exp(-\lambda_{\sum} u) \sum_{j=1}^{3} \lambda_j G_j(t-u, 0) du.$$

These considerations lead to the following *renewal equation* for the reliability function (1):

$$R(t) = H(t) + \int_0^t R(t-\tau) f(\tau) d\tau, \quad t \geq 0.$$

Let us introduce the *renewal density*

$$u(t) = \sum_{k=1}^{\infty} f^{(*k)}(t),$$

where $f^{(*k)}(t)$ is k-th convolution of the function $f(t)$.

Now the solution of the renewal equation can be represented as [6]:

$$R(t) = H(t) + \int_0^t H(t-\tau) u(\tau) d\tau.$$

It is interesting to compare this function with the reliability function of the nonrenewable redundancy system, which is calculated by formula

$$R^*(t) = 1 - \prod_{i=1}^{3} (1 - \exp(-\lambda_i t)).$$

3. Conclusions

The reliability function of the system from three parallel connected renewable components was considered.Our future research will be connected with an investigation of considered system in the random environment [7].

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