UDC 519.2

Stability problems in modern actuarial sciences

E. V. Bulinskaya[*]

[*] *Department of Probability Theory,*
Faculty of Mathematics and Mechanics,
Lomonosov Moscow State University,
Leninskie Gory 1, Moscow, 119234, Russia

Abstract. New models were developed in actuarial sciences during the last two decades. They include different notions of insurance company ruin (bankruptcy) and other objective functions evaluating the company performance. Several types of decision (such as dividends payment, reinsurance, investment) are used for optimization of company functioning. Therefore it is necessary to be sure that the model under consideration is stable with respect to parameters fluctuation and perturbation of underlying stochastic processes. The aim of the talk is description of methods for investigation of these problems and presentation of recent results concerning some insurance models.

Keywords: sensitivity analysis, stability, optimal control, reinsurance, dividends, investment.

1. Introduction

Insurance is a risk-transfer mechanism that ensures full or partial financial compensation for the loss or damage caused by event(s) beyond the control of the insured party. Under an insurance contract, a party (the insurer) indemnifies the other party (the insured) against a specified amount of loss, occurring from specified eventualities within a specified period, provided a fee called premium is paid. Clearly, there arise two cash flows (premiums and indemnities) and mathematical models describing an insurance company performance are of input-output type. The similarity with other applied probability research fields such as queueing, reliability, inventory, finance and many others is quite obvious. Hence, the methods used in one domain may turn out useful in the others.

Actuarial science is the discipline that assesses financial risks in the insurance, finance and other research areas using mathematical and statistical methods, see, e.g., [20]. The history of actuarial sciences is long and interesting, see, e.g., [6]. Here we mention only that its beginning is usually associated with E.Haley's mortality tables which appeared in 1693. The other achievement of the first (deterministic) period is introduction in 1738 of D.Bernoulli's utility functions. The second (stochastic) period is characterized by collective risk theory provided by F.Lundberg in 1903 and further developed by H.Cramér. The third (financial) period has brought the union of stochastic actuarial models with modern finance theory and wide use of high speed computers. This period was very short (not more than 3 decades) compared with two previous. The fourth (modern) period was announced in 2005 by P.Embrechts, see, e.g., [10]. The main feature of this period is emergence of ERM (enterprise risk management) and necessity to deal not only with hazard and financial risks but with operational and strategic risks as well, see [8].

In order to describe any input-output model we need to specify input flow $P(t)$ (premiums), output flow $S(t)$ (claims) and planning horizon $T \le \infty$. Thus, the company capital (surplus, equity) $X(t)$ at time t is given by

$$X(t) = x + P(t) - S(t)$$

where x is the initial capital. The classical Cramér-Lundberg model, as well as the Sparre Andersen one, has a mixed character. That means, the premium is deterministic $P(t) = ct$ where $c > 0$ is a constant premium rate. On the contrary the aggregate claims up to time t are random and have the form $S(t) = \sum_{n=1}^{N(t)} Y_n$. Here Y_n is the nth claim amount whereas $N(t)$ is the number of claims up to time t. For the Cramér-Lundberg model $N(t)$ is a Poisson process with intensity λ while for the Sparre Andersen model $N(t)$ is an ordinary renewal process. In both cases Y_n are nonnegative i.i.d. r.v.'s. For the dual models arising in life-insurance one gets instead of (1) the following relation $X(t) = x - ct + S(t)$.

For optimization of a company performance we have to choose an objective function (criterium, target, risk measure) and define the set of feasible controls (decisions). The most popular approaches are reliability and cost ones. One of the reasons is a two-fold nature of insurance company. At first, there existed only mutual insurance societies aimed at risk transferring and redistribution. Later, the joint stock companies owned by the shareholders began to dominate. So, the primary task of any insurance company is indemnification of its policyholders. That means, the company has to possess enough money to satisfy all the claims. In other words, it is crucial to maximize the non-ruin (or survival) probability, that is, the company reliability. The reliability approach introduced by Cramér and Lundberg is still very popular, see, e.g., [2].

The secondary, but very important, task is dividend payments to shareholders of the company. Due to pioneering paper [11] the cost approach was introduced in the actuarial sciences in the middle of the last century. Since then the expected discounted dividends until ruin is a widely used objective function which has to be maximized, see, e.g. [3]. The minimization of costs entailed by bank loans and inflation was considered in [5] for discrete-time insurance models.

We mention in passing a so-called Gerber-Shiu function estimating ruin severity and its generalizations, see, e.g., [4]. The use of such functions demonstrates the unification of reliability and cost approaches.

The other problems interesting for any insurance company are the choice of underwriting policy, premium calculation principles and reserves to ensure the company solvency, see, e.g., [18]. Moreover, very important decisions are dividend payments, reinsurance and investment.

2. Main results

Consideration of solvency problems, see, e.g., [22], gave rise to new ruin notions such as Parisian ruin, absolute ruin and Omega models. Due to their practical importance, these problems have attracted growing attention in risk theory.

Parisian type ruin will occur if the surplus falls below a prescribed critical level (red zone) and stays there for a continuous time interval of length d. In some respects, this might be a more appropriate measure of risk than classical ruin as it gives the office some time to put its finances in order, see, e.g., [9, 17]. Another type of Parisian ruin includes a stochastic delay (clock) in bankruptcy implementation, see, e.g., [15]. These two types of Parisian ruin start a new clock each time the surplus enters the red zone, either deterministic or stochastic. Proposed in [14] the third type of Parisian ruin (called cumulative) includes the race between a single deterministic clock and the sum of the excursions below the critical level.

One of the first papers treating the absolute ruin is [13]. When the surplus is below zero or the insurer is on deficit, the insurer could borrow money at a debit interest rate to pay claims. Meanwhile, the insurer will repay the debts from the premium income. The negative surplus may return to a positive level. However, when the negative surplus is below a certain critical level, the surplus is no longer able to become positive. Absolute ruin occurs at this moment, see, e.g., [12].

In the Omega model, there is a distinction between ruin (negative surplus) and bankruptcy (going out of business). It is assumed that even with a negative surplus, the company can do business as usual and continue until bankruptcy occurs. The probability for bankruptcy is quantified by a bankruptcy rate function $\omega(x)$, where x is the value of the negative surplus. The symbol for this function leads to the name Omega model, see, e.g., [1].

The first aim of presentation is to carry out asymptotic analysis and optimization of some models of the described above type. In particular, we introduce a new indicator of insurance company performance, namely, the first time η_l^X when the interval of the surplus staying above zero (before the Parisian ruin) becomes greater than l. Then for the Cramér-Lundberg case the explicit form of the Laplace transform of η_l^X is calculated as a function of the model's parameters.

The second aim is to study the systems stability with respect to underlying processes perturbations and parameters fluctuations. Under assumption that claim amounts have exponential distribution with parameter α we perform the sensitivity analysis of the probability of Parisian ruin with a deterministic clock d. For this purpose we use some local and global methods gathered in [21]. Thus, we begin by calculating the partial derivatives with respect to all the parameters α, λ, x, c and d. Then the scatterplots were obtained by Monte-Carlo simulation of ruin probability.

For the Omega model dual to the Cramér-Lundberg insurance model (according to [16]) the expected discounted dividends under barrier strategy can be obtained as the solutions of integro-differential equation. If the claim amounts have the exponential distribution it can be reduced to a second order differential equation. So it is possible to obtain the conditions of Lyapunov stability of the solutions. Another approach used for establishing stability of these models with respect to distributions perturbations is application of probability metrics according to [19].

For the discrete-time models we proceed along the same lines as in [7].

3. Conclusions

We have briefly discussed the new models which arose during the last two decades and problems important for their applications. Due to space limitation it turned out impossible even to formulate precisely the results obtained or provide the numerical results and graphics.

Acknowledgments

The work is partially supported by RFBR grant No 17-01-00468.

References

1. *Albrecher H., Gerber H.U., Shiu E.S.W.* The optimal dividend barrier in the Gamma-Omega model// European Actuarial Journal. — 2011. — Vol. 1. — P. 43–55.
2. *Asmussen S. and Albrecher H.* Ruin probabilities. Second Edition. — World Scientific, New Jersey, 2010.
3. *Avanzi B.* Strategies for dividend distribution: A review // North American Actuarial Journal. — 2009. — Vol. 13, no. 2. — P. 217–251.
4. *Breuer L. and Badescu A.* A generalised Gerber-Shiu measure for Markov-additive risk processes with phase-type claims and capital injections // Scandinavian Actuarial Journal. — 2014. — Vol. 2014, no. 2. — P. 93–115.
5. *Bulinskaya E. V.* On a cost approach in insurance // Review of Applied and Industrial Mathematics. — 2003. — Vol. 10, no. 2. — P. 376–386.
6. *Bulinskaya E.* New research directions in modern actuarial sciences / Modern problems of stochastic analysis and statistics – Festschrift in honor of Valentin Konakov (ed. V.Panov). Springer, 2017, In press.
7. *Bulinskaya E. and Gusak J.* Optimal Control and Sensitivity Analysis for Two Risk Models // Communications in Statistics – Simulation and Computation. — 2016. — Vol. 45, no. 5. — P. 1451–1466.
8. *Cruz M.G., Peters G.W., Shevchenko P.V.* Fundamental Aspects of Operational Risk and Insurance Analytics: A Handbook of Operational Risk. — Wiley, 2015.

9. *Czarna I. and Palmowski Z.* Ruin probability with Parisian delay for a spectrally negative Lévy risk process // Journal of Applied Probability. — 2011. — Vol. 48. — P. 984–1002.
10. *D'Arcy S.P.* On Becoming An Actuary of the Fourth Kind // Proceedings of the Casualty Actuarial Society. — 2005. — Vol. 177. — P. 745–754.
11. *De Finetti B.* Su un'impostazione alternativa della teoria collettiva del rischio // Transactions of the XV-th International Congress of Actuaries. — 1957. — Vol. 2. — P. 433–443.
12. *Fu D., Guo Y.* On the Compound Poisson Model with Debit Interest under Absolute Ruin // International Journal of Science and Research (IJSR). — 2016. — Vol. 5, no. 6. — P. 1872–1875.
13. *Gerber H.U.* Der Einfluss von Zins auf die Ruinwahrscheinlichkeit // Mitteilungen der Vereinigung schweizerischer Versicherungsmathematiker. — 1971. — Vol. 71, no. 1. — P. 63–70.
14. *Guérin H., Renaud J.-F.* On Distribution of Cumulative Parisian Ruin // arXiv:1509.06857v1 [math.PR] 23 Sep 2015.
15. *Landriault D., Renaud J.-F. and Zhou X.* Insurance risk models with Parisian implementation delays // Methodology and Computing in Applied Probability. — 2014. — Vol. 16, no. 3. — P. 583–607.
16. *Liu D. and Liu Z.* Dividend Problems with a Barrier Strategy in the Dual Risk Model until Bankruptcy // Journal of Applied Mathematics. — 2014. — Vol. 2014, Article ID 184098.
17. *Lkabous M.A., Czarna I., Renaud J.-F.* Parisian Ruin for a Refracted Lévy Process // arXiv: 1603.09324v1 [math.PR] 30 March 2016.
18. *Quang P.D.* Ruin Probability in a Generalized Risk Process under Interest Force with Homogenous Markov Chain Premiums // International Journal of Statistics and Probability. — 2013. — Vol. 2, no. 4. — P. 85–92.
19. *Rachev S.T., Klebanov L., Stoyanov S.V., Fabozzi F.* The Methods of Distances in the Theory of Probability and Statistics. — Springer-Verlag, New York, 2013.
20. *Rachev S.T., Stoyanov S.V. and Fabozzi F.J.* Advanced Stochastic Models, Risk Assessment, Portfolio Optimization. — J.Wiley and Sons, Hoboken, New Jersey, 2008.
21. *Saltelli A., Ratto M., Campolongo T., Cariboni J., Gatelli D., Saisana M. and Tarantola S.* Global Sensitivity Analysis. The Primer. — Wiley, 2008.
22. *Sandström A.* Handbook of Solvency for Actuaries and Risk Managers.: Theory and Practice. — Chapman and Hall/CRC Press, 2011.