UDC 519.218.3, 519.23

Adaptive Splitting Method for Failure Estimation in Controllable Degradation System

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Abstract. The work is devoted to the problem of estimating the probability of failure which is regarded as a rare event in the system with gradual and instantaneous failures. Also consider a regenerative degradation process whose characteristics must be obtained by simulation as well. The main intention of this work is to apply the modification of special speed-up technique (adaptive splitting method) to obtain the required point and interval estimates.

Keywords: rare event simulation, adaptive splitting method, degradation process, failure.

1. Introduction

This work deals with the splitting method for effective estimation of the failure probability and other characteristics of a degradation process composed by a sum of successive phases. If the degradation process is Markovian, then some parameters may be calculated analytically. But a problem arises for a non-Markov process. To this end, in more general cases we need to use simulation. We propose the adaptive algorithm for degradation process which allows to solve the problem of setting the number of levels and retrials and how the levels should be chosen before starting the simulation. The proposed splitting method provides a good approximation of the analytical results, where they are available.

2. Failure estimation problem for degradation process

Consider the main notation associated with the degradation process $X := \{X(t)\}_{t\geq 0}$ with a finite state space $E = \{0, 1, \dots, L, \dots, M, \dots, K; F\}$ illustrated in the figure Fig. 1. By construction the process X has a regenerative structure. We will consider the regeneration points $\{\tau_n\}$ as the

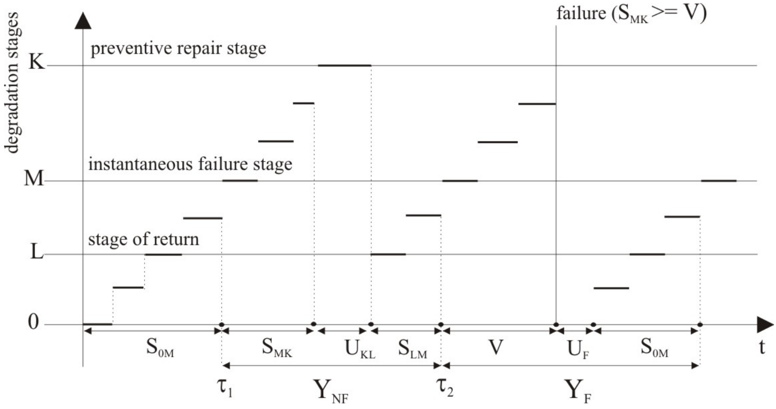

Figure 1. Realization of the degradation process with two types of cycle

return times to state M. Denote by T_i the transition time from state i to $i+1, i \in E \setminus \{K, F\}$. We assume T_i to be independent. Then

$$S_{i\,j} = \sum_{k=i}^{j-1} T_k,\ 0 \le i \le K-1,\ j > i,$$

is the transition time from state j to state i. After the process X hits state M, an instantaneous failure may happen during period V, otherwise, a preventive repair occurs during time S_{MK}. Thus, there are two types of the regeneration cycles of the process X.

Denote by $Y_F = V + U_F + S_{0M}$ the length of the (generic) regeneration cycle with a failure, and $Y_{NF} = S_{MK} + U_{KL} + S_{LM}$ the length of a cycle ended by the maintenance repair. An unconditional (generic) length of a regeneration cycle Y is

$$Y = Y_F \cdot I_{\{V \le S_{MK}\}} + Y_{NF} \cdot I_{\{S_{MK} < V\}},$$

where I denotes indicator function.

The main task is to estimate the probability of failure during a regeneration cycle

$$p_F = \mathbb{P}(S_{MK} \ge V) = \int_0^\infty (1 - F_{MK}(t))dF_V(t),$$

where $F_{MK}(t) = \mathbb{P}(S_{MK} \le t)$ and $F_V(t) = \mathbb{P}(V \le t)$.

3. Conclusions

According to [1] we assume that failure is a rare event so the crude Monte Carlo simulation turns out to be time-consuming.

In [2] we have previously used the modification of multilevel regenerative splitting to estimate a small failure probability related to a degradation process. Furthermore, the dynamics of the degradation process is highly adapted to the application of this method.

Experimential results show that the proposed method gives a significant reduction of simulation time in comparison with the Monte-Carlo simulation and also provide a good approximation of the analytical results.

However, the question of determining the thresholds and splitting factors is still the open problem. The choice of these parameters strongly affects the efficiency of the estimation. To solve this problem we propose to modify the adaptive splitting method (see in [3]) and apply it for the case when the rare threshold for S_{MK} is a random variable. This situation is not typical for rare event problems solved by splitting method, since the rare threshold must be fixed.

Acknowledgments

The work is supported by the RFBR, projects 15-07-02341, 15-07-02354, 15-07-02360, 16-37-60072, 16-37-60072 mol_a_dk. The work was supported by the Ministry of Education of the Russian Federation (the Agreement number 02.a03.21.0008 of 24 June 2016).

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