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Leibniz's Contributions to Financial and Insurance Mathematics

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Abstract. Leibniz was a philosopher who devoted his legal knowledge and his mathematical competence to the service of public welfare. Five aspects of this service will be discussed: 1. Leibniz emphasized the need for the creation of a system of public insurances that was based on the principle of solidarity. 2. He taught how to calculate the cash value of a sum of money that is to be paid in the future. 3. Leibniz acknowledged the importance of statistics for the sake of good governance of a state. But he used strongly simplifying hypotheses for his mathematical model of human life. 4. Leibniz discussed different types of life annuities and deduced the purchase price of a pension by means of his operation of rebate. He found out the presumable life spans of three different types of associations. 5. He explained how life annuities were suitable for eliminating excessive indebtedness of states.

Keywords: public welfare, public insurances, cash value, statistics, mathematical model, life annuities, operation of rebate, associations, indebtedness of states.

1. Introduction

Leibniz combined mathematics, law, and politics when he occupied himself with problems of great public interest that have remained topical until today: 1. Insurance cover, 2. Justice in financial operations, 3. Demographic evolution, 4. Old-age pensions, 5. Public indebtedness. Thus he devoted his legal knowledge and his mathematical competence to the service of the commune bonum or "public welfare". He published nearly nothing of such studies and considerations during his lifetime.

2.1. The economy and science: mathematics as a cultural force

In his memoranda for the Hannoverian duke John Frederick, for the Brandenburg elector Fredrick III in Berlin, and for the German emperor Leopold I in Vienna, he emphasized the need for the creation of a system of public insurance in the interest of a flourishing community and thus in the interest of all, including the sovereign [6, nos. I,1; I,2; I,3; I,4; I,5]. Its purpose was to protect the individual citizen against damages particularly caused by fire or water, "because", he added, "one cannot demand something from people which they do not have" [6, p. 13].

In a memorandum for the foundation of an academy of sciences, written on the 26th of March 1700, he emphasized: "One of the best useful things for the benefit of the country and of the people would be a reliable institution for the protection against damages caused by fire, because in the meantime one has found excellent means against that based on machines and on a mathematical foundation [...] Similarly it would be necessary to establish an institution against damages caused by water [...] To that important end one has but to correctly use geometry. Indeed, now the art of the spirit level has been much advanced." [6, p. 25].

2.2. Negotium mathematici iuris: mathematics as a legal force

How should one calculate the current value of a sum of money that is to be paid in the future? This is a problem that concerns law, politics, and mathematics. The rebate must be determined. None of these three disciplines can decide this question by itself. The just value must favour neither the debtor nor the creditor. It must conciliate the interest within the framework of commercial law and valid law of contract: No composite interest; the legal rate of interest is 5%.

According to civil law the following principle was valid: Somebody who pays earlier than he is obliged to pay, has to pay less at that moment. The legitimate rebate was called *interusurium*, "interest accruing in the meantime". This notion was introduced, but not defined by the Roman law. There was no explanation of how to calculate it, either. When Leibniz applied it to the restitution of debts, to sales by auctions, and to various kinds of insurance (old-age-insurance, etc.) he had to find his own solution because the solutions of Benedict Carpzov and of the jurists contradicted his principles of justice.

Let p be the sum of the lent money, let a be the number of years after which the sum has to be repaid, let i be the legal rate of interest and x the current value looked for, $\nu = 100/i$. Then

$$x = p\left(\frac{\nu}{\nu+1}\right)^a .$$

Leibniz deduced this solution in three ways: as the sum of an infinite series, by stepwise calculating the infinite number of virtual anticipations and compensations, and by inverting the formula of compound interest. But why was the objection to the application of compound interest not justified here? Leibniz answered [6, p. 242f.]: "One can claim interest on interest paid before the date agreed upon. One cannot claim interest on interest which the debtor did not pay punctually."

2.3. Calculus politicus: demography

Leibniz invariably underlined the importance of statistics relating to the country and the people for the sake of good governance of the state.

In 1682 he enumerated 56 questions relevant to his demographical interest [6, no. III.15]. But he preferred hypothetical considerations in order to calculate life expectancy and the value of life annuities. Thus he was a pioneer of mathematical modelling of reality and was conscious of working with strongly simplifying hypotheses. He nearly always used the following assumptions [6, p. 416- 419, 472f.]:
Assumption 1: All people are equally vital.
Assumption 2: Every age is equally fatal.
Assumption 3: The limit of human life is 80 (70, 81) years.

2.4. Life annuities: mathematics as a political force

Leibniz looked for the just price of a life annuity. The duration of life can only be revealed by a prophet, by Divine Revelation. As an actuary he must use the calculus of probabilities in order to attribute a presumable duration to life annuities and thus a just purchase price. He calculated this price of a pension by means of his operation of rebate. It was a matter of the current values of payments made at different times for a common date of purchase.

Let a, i, v have the meaning as above, let x be the purchase price, p the annual pension. The price will be the sum of a geometric series:

$$x = \left(1 - \left(\frac{\nu}{\nu + 1}\right)^a\right) \nu p.$$

Leibniz stepwise generalized the conditions. Originally all pensions are equal. The payments are made after one year. The money is given to one person. If it is the matter of several persons, these persons are of the same age.

Then the conditions are changed: The pensions are unequal. The time intervals between the payments are shorter than one year. The money is given to associations with members who might be of different ages. He called life annuities of the last case "the apogee of this study" [6, p. 468f.]. In the case of associations he needed two definitions:
Definition 1. The life span of an association is the upper limit of the individual life spans of its members. An association survives up to the death of its last member.
Definition 2. The presumed life span of an association of n arbitrary persons is the arithmetical mean of the life spans of n-tuples.

Leibniz determined the life expectations of a group of the same age as well as those of persons of different ages. His combinatorial approach is based on the enumeration of cases. The presupposed conditions are decisive. The persons might have different or equal life spans. Once one has calculated the presumed life span of such an association one has to insert it into the formula for the price of a life annuity. There the calculated value has to replace a.

If for example n persons of a group have the same age, but different life spans and if $x = 80$ is the maximal life span, Leibniz needs four steps in order to deduce the presumed life span of such an association: He looks for all possible associations (combinations) of k persons (of k-tuples). He determines the life spans of the associations (combinations) (pairs, triples,…, n-tuples). He calculates the total number of years of the life spans. He calculates the presumed life span of k persons. His result reads:

$$\frac{80n-1}{n+1}$$

years.

2.5. Public indebtedness

For Leibniz, life annuities, or other amortizable pensions, seems to be the appropriate means for eliminating excessive indebtedness of states or for providing the necessary money for cities, states, and sovereigns. The aim of such an action must be justice. He explicitly explains that public welfare is more important than individual welfare. While we cannot compel an individual against his will to accept a pension, that is, an instalment, so that the debtor can settle his debt, a state which got into financial straits must have this right [6, p. 384f.].

In fact, in case of need and for reasons of equity one might concede a higher percentage than that dictated by mathematics. One has to reckon with, so to speak, a payment of damages. For mathematical reasons, about 6% would be reasonable. For political rather than legal reasons, one could concede 10% or even 14% in order to grant compensation for a risk that is hardly calculable for a private creditor.

Leibniz discusses the example of a city whose revenues are 24000. It loses 5000 because of interest and spends 20000 for public responsibilities [6, p. 386f.]. In order to settle this difficulty, Leibniz suggested financial support for a period of ten years to be paid by the citizens and a temporary restriction of public expenses. In this case, the creditor could get from 13000 to 15000 a year. After ten years the debts would be redeemed.

3. Conclusions

In 1997 Walter Hauser published his PhD dissertation *On the origins of the calculus of probabilities* [1]. He amply discussed the pioneer works by Jan de Witt, John Graunt, William Petty on political arithmetic, on the order of mortality, on demography, on life annuities, on insurance problems which Leibniz knew , cited, and used. He did not say anything about the relevant Leibnizian works. Apart from Parmentier's booklet [5], which was used by Mora Charles [9], most of these works had not been published at that time.

Leibniz's only publication on this subject was and remained his article [4]. Financial and insurance mathematics are an especially good example for his statement that he wrote to the jurist Vincentius Placcius on February 21, 1696 [8, p. 139]: "Qui me non nisi editis novit, non novit." (Who knows me only by my publications, does not know me).

Since then the situation has changed completely. The bilingual volume containing Leibniz's fifty most important papers dealing with this subject appeared in 2000 [6]. The present article is largely based on that volume. In 2001 twenty-five studies were reprinted in [7]. Also in 2001 Jean-Marc Rohrbasser and Jacques Veron published their booklet [10] in Paris. It demonstrates the quick reception of, and the great interest in, these Leibnizian studies.

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