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Transient Analysis of a Multi-Server Queuing Model with Discouraged Arrivals and Retention of Reneging Customers

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Abstract. In this paper, we study a finite capacity Markovian multi-server queuing system with discouraged arrivals, reneging, and retention of reneging customers. The transient state probabilities of the queuing system are obtained by using a computational technique based on the 4th order Runge- Kutta method. With the help of the transient state probabilities, we develop some important measures of performance of the system, such as time-dependent expected system size, time-dependent expected reneging rate, and time-dependent expected retention rate. The transient behavior of the system size probabilities and the expected system size is also studied. Further, the variations in the expected system size, the expected reneging rate, and the expected retention rate with respect to the probability of retaining a reneging customer are also studied. Finally, the effect of discouraged arrivals in the same model is analyzed.

Keywords: transient analysis, reneging, discouraged arrivals, multi-server queuing system, retention.

1. Introduction

Queuing systems are used in the design and analysis of computer-communication networks, production systems, surface and air traffic systems, service systems etc. Queuing systems with customers' impatience have attracted a lot of attention because impatience leads to loss of potential customers. Pioneer works in queuing theory involving impatient customers include that of Haight (1959), and Ancker and Gafarian [(1963a), (1963b)]. Queuing systems with discouraged arrivals are widely studied due to their significant role in managing daily queueing situations. In many practical situations, the service facility possesses defense mechanisms against long waiting lines. For instance, the congestion control mechanism prevents the formation of long queues in computer and communication systems by controlling the transmission rates of packets based on the queue length(of packets) at source or destination. Moreover, a long waiting line may force the servers to increase their rate of service as well as discourage prospective customers which results in balking. Hence, one should study queueing systems by taking into consideration the state-dependent nature of the system. In state-dependent queues the arrival and service rates depend on the number of customers in the system. The discouragement

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affects the arrival rate of the queueing system. Customers arrive in a Poisson fashion with rate that depends on the number of customers present in the system at that time i.e. $\frac{\lambda}{n+1}$. Morse (1958) considers discouragement in which the arrival rate falls according to a negative exponential law.

Queuing systems with customers' impatience have negative impact on the performance of the system, because it leads to the loss of potential customers. Kumar and Sharma (2012a) take this practically valid aspect into account and study an $M/M/1/N$ queuing system with reneging and retention of reneging customers. Kumar (2013) obtains the transient solution of an $M/M/c$ queue with balking, reneging and retention of reneging customers. Kumar and Sharma (2012b) study a finite capacity multi server Markovian queuing model with discouraged arrivals and retention of reneging customers. They derive steady-state solution of the model.

The steady-state results do not reveal the actual functioning of the system. Moreover, stationary results are mainly used within the system design process and it cannot give insight into the transient behavior of the system. That is why, we extend the work of Kumar and Sharma (2012b) in the sense that the transient analysis of the model is performed. The transient numerical behavior is studied by using a numerical technique Runge-Kutta method.

2. Queuing Model Description

The queuing model is based on following assumptions:

1. The customers arrive to the queueing system according to a Poisson process with parameter λ. A customer finding every server busy arrive with arrival rate that depends on the number of customers present in the system at that time i.e. if there are n ($n > c$) customers in the system, the new customer enters the system with rate $\frac{\lambda}{n-c+1}$.
2. There are c servers and the service time distribution is negative exponential with parameter μ. The mean service rate is given by $\mu_n = \{n\mu; 0 \leq n \leq c-1 \text{ and } c\mu; n \geq c\}$.
3. Arriving customers form a single waiting line based on the order of their arrivals and are served according to the first-come, first-served (FCFS) discipline.
4. The capacity of the system is finite (say N).
5. A queue gets developed when the number of customers exceeds the number of servers, that is, when $n > c$. After joining the queue each customer will wait for a certain length of time T (say) for his service to begin. If it has not begun by then he may get renege with probability p and may remain in the queue for his service with probability q $(= 1-p)$ if certain customer retention strategy is used. This time T is a random variable which follows negative exponential distribution with parameter ξ. The reneging rate is given by

$$\xi_n = \begin{cases} 0, & 0 < n \le c \\ (n-c)\xi, & n \ge c+1 \end{cases}$$

6. Initially there is no customer in the system.

Let $P_n(t), n \ge 0$ be the probability that there are n customers in the system at time t.
The differential-difference equations of the model are:

$$\begin{aligned}
\frac{dP_0(t)}{dt} =& -\lambda P_0(t) + \mu P_1(t) \\
\frac{dP_n(t)}{dt} =& -(\lambda + n\mu) P_n(t) + \lambda P_{n-1}(t) + (n+1)\mu P_{n+1}(t), 1 \le n < c \\
\frac{dP_n(t)}{dt} =& -\left[\left(\frac{\lambda}{n-c+2}\right) + c\mu + (n-c)\xi p\right] P_n(t) + \\
& + \left(\frac{\lambda}{n-c+1}\right) P_{n-1}(t) + (c\mu + (n-c+1)\xi p) P_{n+1}(t), \\
& c \le n < N \\
\frac{dP_N(t)}{dt} =& \left(\frac{\lambda}{N-c+1}\right) P_{N-1}(t) - (c\mu + (N-c)\xi p) P_N(t)
\end{aligned}$$

3. Transient analysis of the model

In this section, we perform the transient analysis of a finite capacity multi-server Markovian Queuing model with discouraged arrivals and retention of reneging customers using Runge-Kutta method of fourth order (R-K 4). The "ode45" function of MATLAB software is used to find the transient numerical results corresponding to the differential-difference equations of the model.

We study the following performance measures in transient state:

1. Average Reneging Rate $(R_r(t)) = \sum_{n=c}^{\infty} (n-c)\xi p P_n(t)$
2. Average Retention Rate $(R_R(t)) = \sum_{n=c}^{\infty} (n-c)\xi q P_n(t)$

Now, we perform the transient numerical analysis of the model with the help of a numerical example. We take $N = 10$, $\lambda = 3, \mu = 5, \xi = 0.1, p = 0.4$ and $c = 3$. The results are presented in the form of Figures 1-3. Following are the main observations:

(a) In Figure 1, the probabilities of number of customers in the system at different time points are plotted. We observe that the probability values $P_1(t)$, $P_2(t)$, ..., $P_{10}(t)$ increase gradually until they reach stable values except the probability curve $P_0(t)$ which decreases rapidly in the beginning and then attains steady-state with the passage of time.

(b) The variation in average retention rate with probability of retention is shown in Figure 2. We can see that there is a proportional increase in $R_R(t)$ with increase in q, which justifies the functioning of the model.

(c) In figure 3, the impact of discouraged arrivals on the performance of the system is shown. We compare two multi server finite capacity Markovian queuing systems having retention of reneging customers with and without discouraged arrivals. One can see from Figure 3 that the expected system size is always lower in case of discouraged arrivals as compare to the queuing model without discouragement.

4. Conclusions

The transient analysis of a multi-server queuing system with discouraged arrivals, reneging and retention of reneging customers is performed by using Runge Kutta method. The numerical results are computed with the help of MATLAB software. The effect of probability of retaining a reneging customer on various performance measures is studied. We also study the impact of discouraged arrivals on the system performance.

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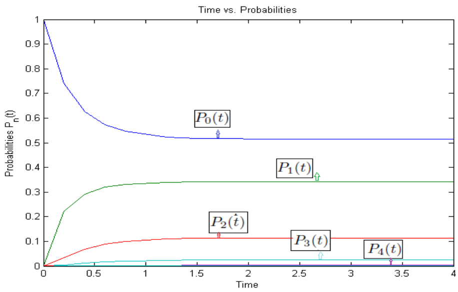

Figure 1. The probability values for different time points are plotted for the case $\lambda = 3, \mu = 5, \xi = 0.1, p = 0.4$ and $c = 3$

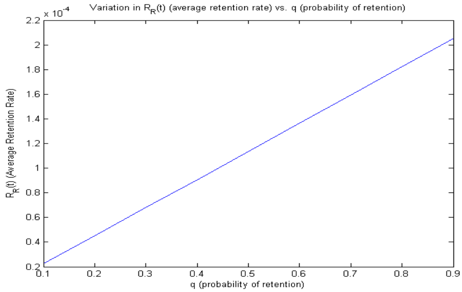

Figure 2. Variation of average retention rate with the variation in probability of retention for the case $\lambda = 3, \mu = 5, \xi = 0.1, p = 0.4, c = 3, t = 0.8$, and $q = 0.1, 0.2, \ldots, 0.9$

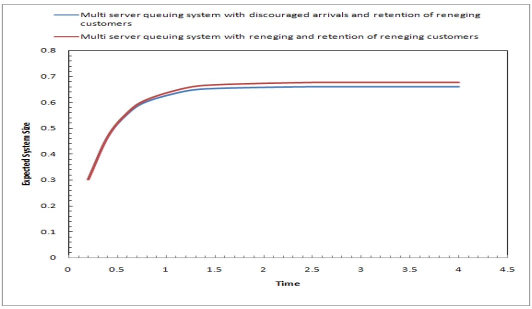

Figure 3. The impact of discouragement on expected system size